# Why

# —

# an intermediate language for deductive program verification

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# Motivations

how to do **deductive program verification** on realistic programs?

- deductive verification means that we want to prove safety but also behavioral correctness, with arbitrary proof complexity
- realistic programs means pointers, aliases, dynamic allocation, arbitrary data structures, etc.

# Motivations

since **Hoare logic** (1968), we know how to turn a program correctness into logical formulas, the so-called verification conditions

we could

- design Hoare logic rules for a real programming language
- choose an interactive theorem prover

**the Why approach:** don't do that!

# Principles

instead,

- design a **small** language dedicated to program verification and compile complex programs into it
- use as **many theorem provers** as possible (interactive or automatic)

# Related Work

there is another such tool: the **Boogie** tool developed at Microsoft Research, initially in the context of the SPEC# project (Barnett, Leino, Schulte)

there are differences but the main idea is the same: verification conditions should be computed on a small, dedicated language

# Overview

1. the Why language

   and its application to the verification of algorithms

2. Why as an intermediate language for program verification

   complete example with a C program

# The essence of Hoare logic

the essence of Hoare logic fits in the rule for assignment

$$\frac{}{\{P[x \leftarrow E]\}\ x := E\ \{P\}}$$

two key ideas

- there is **no alias**, since only variable $x$ is substituted
- the **pure** expression $E$ belongs to both **logic and program**

# The essence of Hoare logic

WHY captures these ideas

- programs can manipulate pure values (i.e. logical terms) arbitrarily
- the sole data structures are mutable variables containing pure values
- any program that would create an alias is rejected

# Structure of a Why File

a Why file contains

- logical declarations

```
logic a : int
logic f : int, int -> int
axiom A : forall x: int. ...
type set
```

- variable/program declarations

```
parameter x : int ref
parameter p1 : a: int -> ...
```

- program implementations

```
let p2 (x: int) (y: int) = ...
```

# Predefined Types

a few types and symbols are predefined

- a type `int` of arbitrary precision integers, with usual infix syntax
- a type `real` of real numbers
- a type `bool` of booleans
- a singleton type `unit`

# ML Syntax

one nice idea taken from functional programming:
no distinction between expressions and statements

$\Rightarrow$ less constructs, thus less rules
$\Rightarrow$ side-effects in expressions for free

but Why is not at all a functional language

# A First Example

let us check that $n$ is even with the following (rather stupid) code

$$
\begin{array}{l}
\text{while } n \geq 2 \\
\quad n \leftarrow n - 2 \\
\text{return } n = 0
\end{array}
$$

# A First Example

we first introduce the predicate `even`, as an uninterpreted predicate with two axioms

```
logic even : int -> prop

axiom even0 :
  even(0)

axiom even2 :
  forall n: int.  n >= 0 -> even(n) -> even(n+2)
```

# A First Example

the program `is_even` is a function with `n` as argument

its body is a Hoare triple

```
let is_even (n: int) =
  { n >= 0 }
  ...
  { result=true -> even(n) }
```

in the postcondition, `result` is the returned value, i.e. the value of the function body (which is an expression)

# A First Example

we introduce a local mutable variable x initialized to n

```
let is_even (n: int) =
  { n >= 0 }
  let x = ref n in
  ...
  { result=true -> even(n) }
```

# A First Example

finally, we add the `while` loop and its invariant

```
let is_even (n: int) =
  { n >= 0 }
  let x = ref n in
  while !x >= 2 do
    { invariant even(x) -> even(n) }
    x := !x - 2
  done;
  !x = 0
  { result=true -> even(n) }
```

# A First Example

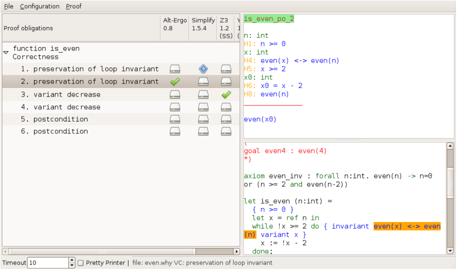
we are ready for program verification

two options

- command line tool

```
why --smtlib even.why
why --pvs even.why
```

- GUI to display verification conditions and launch provers



# A First Example

termination can be proved by adding a **variant** to the loop annotation

```
let is_even (n: int) =
  { n >= 0 }
  let x = ref n in
  while !x >= 2 do
    { invariant even(x) -> even(n)
      variant x }
    x := !x - 2
  done;
  !x = 0
  { result=true -> even(n) }
```

# A First Example

to get completeness, we add the axiom

```
axiom even_inv :
  forall n: int.  even(n) -> n=0 or (n >= 2 and even(n-2))
```

and we turn the postcondition (and the invariant) into an equivalence

```
let is_even (n: int) =
  { n >= 0 }
  ...
  { result=true <-> even(n) }
```

# Previous Values of a Variable

a function argument can be a mutable variable

here, it simplifies the code

```
let is_even2 (n: int ref) =
  while !n >= 2 do
    n := !n - 2
  done;
  !n = 0
```

but it complicates the specification, since values of `n` at different program steps are now involved

# Previous Values of a Variable

in a postcondition, `n@` stands for the value of `n` in the pre-state

```
let is_even2 (n: int ref) =
  { n >= 0 }
  ...
  { result=true <-> even(n@) }
```

# Previous Values of a Variable

more generally, a program point can be labelled (like for a `goto`) and then `x@L` stands for the value of `x` at point `L`

here it is used to refer to the value of `n` before the loop

```
let is_even2 (n: int ref) =
  { n >= 0 }
  L:
  while !n >= 2 do
    { invariant even(n) <-> even(n@L) }
    ...
```

# Auxiliary Variables

WHY favors the use of **labels** instead of the traditional **auxiliary variables**, since it simplifies the VCs

note that it is yet possible to use auxiliary variables, if desired: simply add extra arguments to functions

# Recursive Functions

WHY supports recursive functions

```
let rec is_even_rec (n: int) : bool {variant n} =
  { n >= 0 }
  if n >= 2 then is_even_rec (n-2) else n=0
  { result=true <-> even(n) }
```

# Other Features

WHY also features

- **polymorphism**, in both logic and programs
- **exceptions** in programs, and corresponding annotations
- **local assertions**
- **modularity**, i.e. verification only depends on specifications

all of these features are illustrated in the following

# A More Complex Example

let us consider a more complex program: Dijkstra's algorithm for single-source shortest path in a weighted graph

we are going to use WHY to verify the **algorithm** i.e. a high-level pseudo-code, e.g. from the Cormen-Leiserson-Rivest, **not an actual implementation** in a given programming language

# Dijkstra's Shortest Path

single-source shortest path in a weighted graph

$S \leftarrow \emptyset$
$Q \leftarrow \{src\};$
$d[src] \leftarrow 0$
while $Q \backslash S$ not empty do
  extract $u$ from $Q \backslash S$ with minimal distance $d[u]$
  $S \leftarrow S \cup \{u\}$
  for each vertex $v$ such that $u \xrightarrow{w} v$
    $d[v] \leftarrow \min(d[v], d[u] + w)$
    $Q \leftarrow Q \cup \{v\}$

# Dijkstra's Shortest Path: Finite Sets

we need **finite sets** for the program and its specification

- set of vertices $V$
- set of successors of $u$
- sets $S$ and $Q$

all we need is

- the empty set $\emptyset$
- addition $\{x\} \cup s$
- subtraction $s \backslash \{x\}$
- membership predicate $x \in s$

# Dijkstra's Shortest Path: Finite Sets

let us axiomatize polymorphic sets

```
type 'a set

logic set_empty : 'a set
logic set_add : 'a, 'a set -> 'a set
logic set_rmv : 'a, 'a set -> 'a set
logic In : 'a, 'a set -> prop

predicate Is_empty(s : 'a set) =
  forall x:'a. not In(x, s)

predicate Incl(s1 : 'a set, s2 : 'a set) =
  forall x:'a. In(x, s1) -> In(x, s2)
```

# Dijkstra's Shortest Path: Finite Sets

```
axiom set_empty_def :
  Is_empty(set_empty)

axiom set_add_def :
  forall x,y: 'a.  forall s: 'a set.
  In(x, set_add(y,s)) <-> (x = y or In(x, s))

axiom set_rmv_def :
  forall x,y: 'a.  forall s: 'a set.
  In(x, set_rmv(y,s)) <-> (x <> y and In(x, s))
```

# Dijkstra's Shortest Path: the Weighted Graph

the graph is introduced as follows

```
type vertex

logic V : vertex set

logic g_succ : vertex -> vertex set

axiom g_succ_sound : forall x: vertex. Incl(g_succ(x), V)

logic weight : vertex, vertex -> int (* a total function *)

axiom weight_nonneg : forall x,y: vertex. weight(x,y) >= 0
```

# Dijkstra's Shortest Path: Visited Vertices

the set $S$ of visited vertices is introduced as a global variable containing a value of type `vertex set`

```
parameter S : vertex set ref
```

to modify $S$, we could use assignment (`:=`) directly, but we can equivalently declare a function

```
parameter S_add :
  x: vertex -> {} unit writes S { S = set_add(x, S@) }
```

which reads as "function `S_add` takes a vertex `x`, has no precondition, returns nothing, modifies the contents of `S` and has postcondition $S = \texttt{set\_add}(x, S@)$"

# Dijkstra's Shortest Path: the Priority Queue

we proceed similarly for the priority queue

```
parameter Q : vertex set ref

parameter Q_is_empty :
  unit ->
    { }
    bool reads Q
    { if result then Is_empty(Q) else not Is_empty(Q) }

parameter init :
  src: vertex -> {} ...

parameter relax :
  u: vertex -> v: vertex -> {} ...
```

# Dijkstra's Shortest Path: Demo

17 VCs are generated

they are all automatically discharged, with the help of two lemmas

these two lemmas are proved using an interactive proof assistant (they require induction)

**demo**

## using Why as an intermediate language

# Program Verification in the Large

let us say we want to verify programs written in a language such as C or Java; what do we need?

- to cope with complex **data structures** (arrays, pointers, records, objects, etc.) and possible **aliasing**
- to cope with **new control statements** such as `for` loops, abrupt `return`, `switch`, `goto`, etc.
- to cope with memory allocation, function pointers, dynamic binding, casts, machine arithmetic, etc.

# Solutions

WHY can be used conveniently to handle most of these aspects

two connected parts

- we design a **memory model**, that is a set of logical types and operations to describe the memory layout
- we design a **compilation** process to translate programs in WHY constructs

# A Simple Example

let us consider the following C code

```
int binary_search(int* t, int n, int v) {
  int l = 0, u = n-1;
  while (l <= u) {
    int m = (l + u) / 2;
    if (t[m] < v)
      l = m + 1;
    else if (t[m] > v)
      u = m - 1;
    else
      return m;
  }
  return -1;
}
```

# Binary Search

two (simple) problems with this code

- C pointers (but no pointer arithmetic, i.e. arrays)

```
int binary_search(int* t, int n, int v) { ...
```

- an abrupt `return` in the `while` loop

```
while (l <= u) {
  if ...
  else
    return m;
}
```

# Binary Search: Memory Model

we consider a very simple memory model here

```
type pointer

type memory

logic get : memory, pointer, int -> int

parameter mem : memory ref
  (* the current state of the memory *)
```

# Binary Search: Memory Model

some remarks at this point

- we assume the memory to be accessed by words (type `int`); accessing the same portion of memory using a `char*` pointer would require a finer model
- C local variables can be translated into WHY local variables, unless their address is taken

# Binary Search: Memory Model

thus the code looks like

```
let binary_search (t: pointer) (n: int) (v: int) =
  { ... }
  let l = ref 0 in
  let u = ref (n-1) in
  while !l <= !u do
    let m = (!l + !u) / 2 in
    if get !mem t m < v then l := m + 1
    else if get !mem t m > v then u := m - 1
    else ...
  done
  ...
```

# Binary Search: `return` Statement

to interpret the `return` statement we introduce an exception

```
exception Return_int of int
```

the whole function body is put into a `try/with` statement

```
let binary_search (t: pointer) (n: int) (v: int) =
  try
    ...
  with Return_int r ->
    r
  end
```

and any `return e` is translated into

```
  raise (Return_int e)
```

# Binary Search: Demo

with suitable annotations for correctness, completeness and termination, we get 17 VCs

with the help of the axiom

```
axiom mean_1: forall x,y: int.  x <= y -> x <= (x+y)/2 <= y
```

all VCs are discharged automatically

**demo**

# Binary Search: Array Bound Checking

let us say we want to add **array bound checking**

we need to refine our model with a notion of **block size**

```
logic block_size: memory, pointer -> int
```

it is then convenient to introduce a *function* to access memory

```
parameter get_:
  p: pointer -> ofs: int ->
    { 0 <= ofs < block_size(mem, p) }
    int reads mem
    { result = get(mem, p, ofs) }
```

so that its precondition introduces the suitable VC

# Binary Search: Array Bound Checking

we get 2 additional VCs, easily proved once we add the suitable requirement

```
let binary_search (t: pointer) (n: int) (v: int) =
  { n >= 0 and block_size(mem, t) >= n and ...  }
  ...
```

# Binary Search: Machine Arithmetic

finally, let us model **32 bit integers**,

two possibilities

- to prove that there is no arithmetic overflow
- to model modulo arithmetic faithfully

one requirement:
we do not want to loose the arithmetic capabilities of the provers

# Binary Search: Machine Arithmetic

we introduce a new type for 32 bit integers

```
type int32
```

the value of an int32 is given by

```
logic to_int: int32 -> int
```

annotations only use arbitrary prevision integers, i.e.
if $x$ of type `int32` appears in an annotation, it is actually `to_int(`$x$`)`

# Binary Search: Machine Arithmetic

we need to set the range of 32 bit integers

when using them...

```
axiom int32_domain:
  forall x: int32.  -2147483648 <= to_int(x) <= 2147483647
```

... and when building them

```
parameter of_int :
  x: int ->
    { -2147483648 <= x <= 2147483647 }
    int32
    { to_int(result) = x }
```

# Binary Search: Machine Arithmetic

and that's it!

let us prove the absence of integer overflow in binary search

**demo**

# Binary Search: Machine Arithmetic

we found a bug (that is the purpose of verification, after all)

indeed, when computing

```
int m = (l + u) / 2;
```

the addition `l+u` may overflow
(for instance on a 32 bit architecture with arrays of billions of elements)

it can be fixed as follows

```
int m = l + (u - l) / 2;
```

# Conclusion

# Things Not Discussed in that Tutorial

regarding WHY itself

- how to exclude aliases
- how to send VCs to all provers (typing systems differ)
- how to compute VCs efficiently

regarding the use of WHY

- how to design a high-level specification language
- how to design a more subtle memory model (component-as-array, regions, etc.)
- how to model floating-point arithmetic

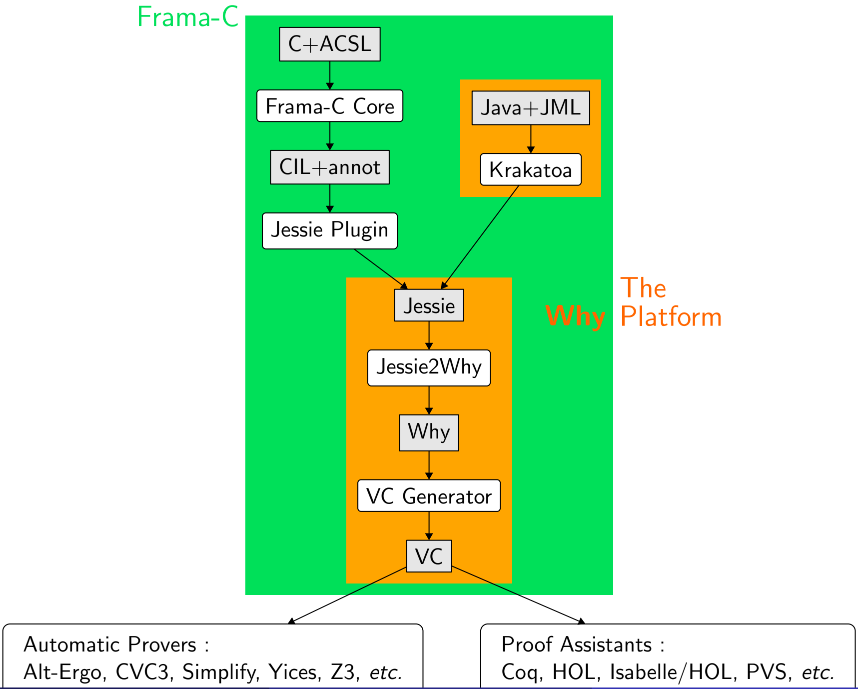
# Existing Software

in the **ProVal** team, we develop the following softwares

- **Jessie**, another intermediate language on top of WHY
- **Krakatoa**, a tool to verify JML-annotated Java programs
- **Alt-Ergo**, an SMT solver with WHY syntax as input

we also collaborate to **Frama-C**, a platform to verify C programs (which subsumes the tool Caduceus formerly developed at ProVal)

our tools deal with **floating-point arithmetic**: annotations, models, interactive and automatic proofs


Frama-C
C+ACSL
Frama-C Core
CIL+annot
Jessie Plugin
Java+JML
Krakatoa
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Why Platform
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Jessie2Why
Why
VC Generator
VC
Automatic Provers :
Alt-Ergo, CVC3, Simplify, Yices, Z3, *etc.*
Proof Assistants :
Coq, HOL, Isabelle/HOL, PVS, *etc.*

thank you