Why
—
an intermediate language for deductive program verification

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Motivations

how to do **deductive program verification** on realistic programs?

- deductive verification means that we want to prove safety but also behavioral correctness, with arbitrary proof complexity
- realistic programs means pointers, aliases, dynamic allocation, arbitrary data structures, etc.

Motivations

since **Hoare logic** (1968), we know how to turn a program correctness into logical formulas, the so-called verification conditions

we could

- design Hoare logic rules for a real programming language
- choose an interactive theorem prover

the Why approach: don't do that!

Principles

instead,

- design a **small** language dedicated to program verification and compile complex programs into it
- use as **many theorem provers** as possible (interactive or automatic)

Related Work

there is another such tool: the **Boogie** tool developed at Microsoft Research, initially in the context of the SPEC# project (Barnett, Leino, Schulte)

there are differences but the main idea is the same: verification conditions should be computed on a small, dedicated language

Overview

1. the WHY language

   and its application to the verification of algorithms

2. WHY as an intermediate language for program verification

   complete example with a C program

The essence of Hoare logic

the essence of Hoare logic fits in the rule for assignment

$$\frac{}{\{P[x \leftarrow E]\}\ x := E\ \{P\}}$$

two key ideas

- there is **no alias**, since only variable x is substituted
- the **pure** expression E belongs to both **logic and program**

The essence of Hoare logic

WHY captures these ideas

- programs can manipulate pure values (i.e. logical terms) arbitrarily
- the sole data structures are mutable variables containing pure values
- any program that would create an alias is rejected

Structure of a WHY File

a WHY file contains

- logical declarations

```
logic a : int
logic f : int, int -> int
axiom A : forall x: int. ...
type set
```

- variable/program declarations

```
parameter x : int ref
parameter p1 : a: int -> ...
```

- program implementations

```
let p2 (x: int) (y: int) = ...
```

Predefined Types

a few types and symbols are predefined

- a type `int` of arbitrary precision integers, with usual infix syntax
- a type `real` of real numbers
- a type `bool` of booleans
- a singleton type `unit`

ML Syntax

one nice idea taken from functional programming:
no distinction between expressions and statements

⇒ less constructs, thus less rules
⇒ side-effects in expressions for free

but WHY is not at all a functional language

A First Example

let us check that *n* is even with the following (rather stupid) code

```
while $n \geq 2$
   $n \leftarrow n - 2$
return $n = 0$
```

A First Example

we first introduce the predicate `even`, as an uninterpreted predicate with two axioms

```
logic even : int -> prop

axiom even0 :
  even(0)

axiom even2 :
  forall n: int.  n >= 0 -> even(n) -> even(n+2)
```

A First Example

the program `is_even` is a function with `n` as argument

its body is a Hoare triple

```
let is_even (n: int) =
  { n >= 0 }
  ...
  { result=true -> even(n) }
```

in the postcondition, `result` is the returned value, i.e. the value of the function body (which is an expression)

A First Example

we introduce a local mutable variable `x` initialized to `n`

```
let is_even (n: int) =
  { n >= 0 }
  let x = ref n in
  ...
  { result=true -> even(n) }
```

A First Example

finally, we add the `while` loop and its invariant

```
let is_even (n: int) =
  { n >= 0 }
  let x = ref n in
  while !x >= 2 do
    { invariant even(x) -> even(n) }
    x := !x - 2
  done;
  !x = 0
  { result=true -> even(n) }
```

A First Example

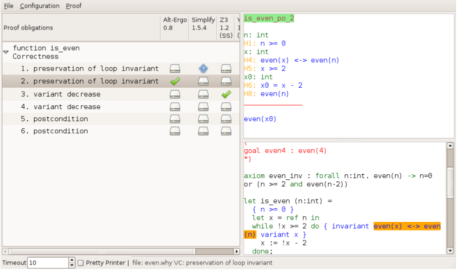
we are ready for program verification

two options

- command line tool

```
why --smtlib even.why
why --pvs even.why
```

- GUI to display verification conditions and launch provers



Proof obligations	Alt-Ergo 0.8	Simplify 1.5.4	Z3 1.2 (SS)
function is_even Correctness			
1. preservation of loop invariant		?	
2. preservation of loop invariant	✓		
3. variant decrease			✓
4. variant decrease			
5. postcondition			
6. postcondition			

```
is_even_po_2

n: int
H1: n >= 0
x: int
H4: even(x) <-> even(n)
H5: x >= 2
x0: int
H6: x0 = x - 2
H8: even(n)
_______________

even(x0)
```

```
(
goal even4 : even(4)
*)

axiom even_inv : forall n:int. even(n) -> n=0
or (n >= 2 and even(n-2))

let is_even (n:int) =
  { n >= 0 }
  let x = ref n in
  while !x >= 2 do { invariant even(x) <-> even
(n) variant x }
    x := !x - 2
  done;
```

Timeout 10 ☐ Pretty Printer | file: even.why VC: preservation of loop invariant

A First Example

termination can be proved by adding a **variant** to the loop annotation

```
let is_even (n: int) =
  { n >= 0 }
  let x = ref n in
  while !x >= 2 do
    { invariant even(x) -> even(n)
      variant x }
    x := !x - 2
  done;
  !x = 0
  { result=true -> even(n) }
```

A First Example

to get completeness, we add the axiom

```
axiom even_inv :
  forall n: int.  even(n) -> n=0 or (n >= 2 and even(n-2))
```

and we turn the postcondition (and the invariant) into an equivalence

```
let is_even (n: int) =
  { n >= 0 }
  ...
  { result=true <-> even(n) }
```

Previous Values of a Variable

a function argument can be a mutable variable

here, it simplifies the code

```
let is_even2 (n: int ref) =
  while !n >= 2 do
    n := !n - 2
  done;
  !n = 0
```

but it complicates the specification, since values of `n` at different program steps are now involved

Previous Values of a Variable

in a postcondition, `n@` stands for the value of `n` in the pre-state

```
let is_even2 (n: int ref) =
  { n >= 0 }
  ...
  { result=true <-> even(n@) }
```

Previous Values of a Variable

more generally, a program point can be labelled (like for a `goto`) and then `x@L` stands for the value of `x` at point `L`

here it is used to refer to the value of `n` before the loop

```
let is_even2 (n: int ref) =
  { n >= 0 }
  L:
  while !n >= 2 do
    { invariant even(n) <-> even(n@L) }
    ...
```

Auxiliary Variables

WHY favors the use of **labels** instead of the traditional **auxiliary variables**, since it simplifies the VCs

note that it is yet possible to use auxiliary variables, if desired: simply add extra arguments to functions

Recursive Functions

WHY supports recursive functions

```
let rec is_even_rec (n: int) : bool {variant n} =
  { n >= 0 }
  if n >= 2 then is_even_rec (n-2) else n=0
  { result=true <-> even(n) }
```

Other Features

WHY also features

- **polymorphism**, in both logic and programs
- **exceptions** in programs, and corresponding annotations
- **local assertions**
- **modularity**, i.e. verification only depends on specifications

all of these features are illustrated in the following

A More Complex Example

let us consider a more complex program: Dijkstra's algorithm for single-source shortest path in a weighted graph

we are going to use WHY to verify the **algorithm** i.e. a high-level pseudo-code, e.g. from the Cormen-Leiserson-Rivest, **not an actual implementation** in a given programming language

Dijkstra's Shortest Path

single-source shortest path in a weighted graph

```
$S \leftarrow \emptyset$
$Q \leftarrow \{src\};$
$d[src] \leftarrow 0$
while $Q \backslash S$ not empty do
  extract $u$ from $Q \backslash S$ with minimal distance $d[u]$
  $S \leftarrow S \cup \{u\}$
  for each vertex $v$ such that $u \xrightarrow{w} v$
    $d[v] \leftarrow \min(d[v], d[u] + w)$
    $Q \leftarrow Q \cup \{v\}$
```

Dijkstra's Shortest Path: Finite Sets

we need **finite sets** for the program and its specification

- set of vertices V
- set of successors of u
- sets S and Q

all we need is

- the empty set $\emptyset$
- addition $\{x\} \cup s$
- subtraction $s \backslash \{x\}$
- membership predicate $x \in s$

Dijkstra's Shortest Path: Finite Sets

let us axiomatize polymorphic sets

```
type 'a set

logic set_empty : 'a set
logic set_add : 'a, 'a set -> 'a set
logic set_rmv : 'a, 'a set -> 'a set
logic In : 'a, 'a set -> prop

predicate Is_empty(s : 'a set) =
  forall x:'a.  not In(x, s)

predicate Incl(s1 : 'a set, s2 : 'a set) =
  forall x:'a.  In(x, s1) -> In(x, s2)
```

Dijkstra's Shortest Path: Finite Sets

```
axiom set_empty_def :
  Is_empty(set_empty)

axiom set_add_def :
  forall x,y: 'a.  forall s: 'a set.
  In(x, set_add(y,s)) <-> (x = y or In(x, s))

axiom set_rmv_def :
  forall x,y: 'a.  forall s: 'a set.
  In(x, set_rmv(y,s)) <-> (x <> y and In(x, s))
```

Dijkstra's Shortest Path: the Weighted Graph

the graph is introduced as follows

```
type vertex

logic V : vertex set

logic g_succ : vertex -> vertex set

axiom g_succ_sound : forall x: vertex. Incl(g_succ(x), V)

logic weight : vertex, vertex -> int (* a total function *)

axiom weight_nonneg : forall x,y: vertex. weight(x,y) >= 0
```

Dijkstra's Shortest Path: Visited Vertices

the set S of visited vertices is introduced as a global variable containing a value of type `vertex set`

```
parameter S : vertex set ref
```

to modify S, we could use assignment (`:=`) directly, but we can equivalently declare a function

```
parameter S_add :
  x: vertex -> {} unit writes S { S = set_add(x, S@) }
```

which reads as "function `S_add` takes a vertex `x`, has no precondition, returns nothing, modifies the contents of `S` and has postcondition $S = \texttt{set\_add}(x, S\texttt{@})$"

Dijkstra's Shortest Path: the Priority Queue

we proceed similarly for the priority queue

```
parameter Q : vertex set ref

parameter Q_is_empty :
  unit ->
    { }
    bool reads Q
    { if result then Is_empty(Q) else not Is_empty(Q) }

parameter init :
  src: vertex -> {} ...

parameter relax :
  u: vertex -> v: vertex -> {} ...
```

Dijkstra's Shortest Path: Demo

17 VCs are generated

they are all automatically discharged, with the help of two lemmas

these two lemmas are proved using an interactive proof assistant (they require induction)

demo

using Why as an intermediate language

Program Verification in the Large

let us say we want to verify programs written in a language such as C or Java; what do we need?

- to cope with complex **data structures** (arrays, pointers, records, objects, etc.) and possible **aliasing**
- to cope with **new control statements** such as `for` loops, abrupt `return`, `switch`, `goto`, etc.
- to cope with memory allocation, function pointers, dynamic binding, casts, machine arithmetic, etc.

Solutions

WHY can be used conveniently to handle most of these aspects

two connected parts

- we design a **memory model**, that is a set of logical types and operations to describe the memory layout
- we design a **compilation** process to translate programs in WHY constructs

A Simple Example

let us consider the following C code

```
int binary_search(int* t, int n, int v) {
  int l = 0, u = n-1;
  while (l <= u) {
    int m = (l + u) / 2;
    if (t[m] < v)
      l = m + 1;
    else if (t[m] > v)
      u = m - 1;
    else
      return m;
  }
  return -1;
}
```

Binary Search

two (simple) problems with this code

- C pointers (but no pointer arithmetic, i.e. arrays)

```
int binary_search(int* t, int n, int v) { ...
```

- an abrupt `return` in the `while` loop

```
while (l <= u) {
  if ...
  else
    return m;
}
```

Binary Search: Memory Model

we consider a very simple memory model here

```
type pointer

type memory

logic get : memory, pointer, int -> int

parameter mem : memory ref
  (* the current state of the memory *)
```

Binary Search: Memory Model

some remarks at this point

- we assume the memory to be accessed by words (type `int`); accessing the same portion of memory using a `char*` pointer would require a finer model
- C local variables can be translated into WHY local variables, unless their address is taken

Binary Search: Memory Model

thus the code looks like

```
let binary_search (t: pointer) (n: int) (v: int) =
  { ...  }
  let l = ref 0 in
  let u = ref (n-1) in
  while !l <= !u do
    let m = (!l + !u) / 2 in
    if get !mem t m < v then l := m + 1
    else if get !mem t m > v then u := m - 1
    else ...
  done
  ...
```

Binary Search: `return` Statement

to interpret the `return` statement we introduce an exception

```
exception Return_int of int
```

the whole function body is put into a `try/with` statement

```
let binary_search (t: pointer) (n: int) (v: int) =
  try
    ...
  with Return_int r ->
    r
  end
```

and any `return e` is translated into

```
  raise (Return_int e)
```

Binary Search: Demo

with suitable annotations for correctness, completeness and termination, we get 17 VCs

with the help of the axiom

```
axiom mean_1: forall x,y: int.  x <= y -> x <= (x+y)/2 <= y
```

all VCs are discharged automatically

demo

Binary Search: Array Bound Checking

let us say we want to add **array bound checking**

we need to refine our model with a notion of **block size**

```
logic block_size: memory, pointer -> int
```

it is then convenient to introduce a *function* to access memory

```
parameter get_:
  p: pointer -> ofs: int ->
    { 0 <= ofs < block_size(mem, p) }
    int reads mem
    { result = get(mem, p, ofs) }
```

so that its precondition introduces the suitable VC

Binary Search: Array Bound Checking

we get 2 additional VCs, easily proved once we add the suitable requirement

```
let binary_search (t: pointer) (n: int) (v: int) =
  { n >= 0 and block_size(mem, t) >= n and ...  }
  ...
```

Binary Search: Machine Arithmetic

finally, let us model **32 bit integers**,

two possibilities

- to prove that there is no arithmetic overflow
- to model modulo arithmetic faithfully

one requirement:
we do not want to loose the arithmetic capabilities of the provers

Binary Search: Machine Arithmetic

we introduce a new type for 32 bit integers

```
type int32
```

the value of an `int32` is given by

```
logic to_int: int32 -> int
```

annotations only use arbitrary prevision integers, i.e.
if x of type `int32` appears in an annotation, it is actually `to_int(`x`)`

Binary Search: Machine Arithmetic

we need to set the range of 32 bit integers

when using them...

```
axiom int32_domain:
  forall x: int32.  -2147483648 <= to_int(x) <= 2147483647
```

... and when building them

```
parameter of_int :
  x: int ->
    { -2147483648 <= x <= 2147483647 }
    int32
    { to_int(result) = x }
```

Binary Search: Machine Arithmetic

and that's it!

let us prove the absence of integer overflow in binary search

demo

Binary Search: Machine Arithmetic

we found a bug (that is the purpose of verification, after all)

indeed, when computing

```
int m = (l + u) / 2;
```

the addition `l+u` may overflow
(for instance on a 32 bit architecture with arrays of billions of elements)

it can be fixed as follows

```
int m = l + (u - l) / 2;
```

Conclusion

Things Not Discussed in that Tutorial

regarding WHY itself

- how to exclude aliases
- how to send VCs to all provers (typing systems differ)
- how to compute VCs efficiently

regarding the use of WHY

- how to design a high-level specification language
- how to design a more subtle memory model (component-as-array, regions, etc.)
- how to model floating-point arithmetic

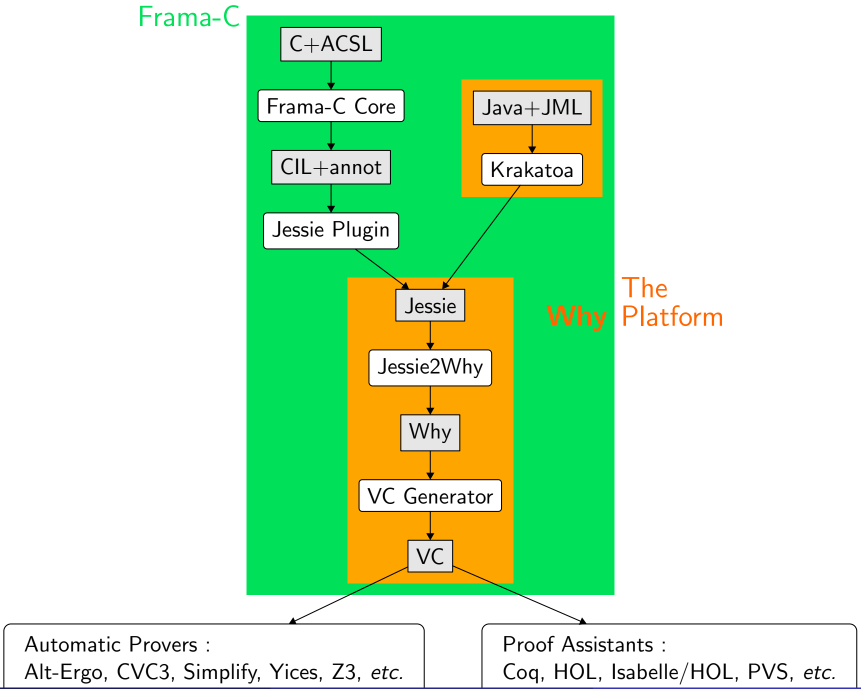
Existing Software

in the **ProVal** team, we develop the following softwares

- **Jessie**, another intermediate language on top of WHY
- **Krakatoa**, a tool to verify JML-annotated Java programs
- **Alt-Ergo**, an SMT solver with WHY syntax as input

we also collaborate to **Frama-C**, a platform to verify C programs (which subsumes the tool Caduceus formerly developed at ProVal)

our tools deal with **floating-point arithmetic**: annotations, models, interactive and automatic proofs



Frama-C

C+ACSL

Frama-C Core

CIL+annot

Jessie Plugin

Java+JML

Krakatoa

Jessie

Jessie2Why

Why

VC Generator

VC

The **Why** Platform

Automatic Provers :
Alt-Ergo, CVC3, Simplify, Yices, Z3, *etc.*

Proof Assistants :
Coq, HOL, Isabelle/HOL, PVS, *etc.*

thank you